Topos-theoretic Model of the Deutsch multiverse

Alexander K. Guts

Department of Computer Science, Omsk State University
644077 Omsk-77 RUSSIA

E-mail: guts@univer.omsk.su

November 21, 2001

ABSTRACT

The Deutsch multiverse is collection of parallel universes. In this article a formal theory and a topos-theoretic model of the Deutsch multiverse are given. For this the Lawvere-Kock Synthetic Differential Geometry and topos models for smooth infinitesimal analysis are used. Physical properties of multi-variant and many-dimensional parallel universes are discussed. Quantum fluctuations of universe geometry are considered. Photon ghosts in parallel universes are found.

This paper was published in Russian journal "Mathematical Structures and Modeling", No.8, 76-90 (2001) (ftp://cmm.univer.omsk.su/pub/sbornik8/guts.zip).

Introduction

In David Deutsch 's book [1] the sketch of structure of physical reality named Multiverse which is set of the parallel universes is given. Correct description of the Multiverse (as Deutsch considers) can be done only within the framework of the quantum theory.

In this article a formal theory and a topos-theoretic model of the Deutsch multiverse are given.

We wish to preserve the framework of the mathematical apparatus of the 4-dimensional General theory of the relativity, and so we shall consider the Universe as concrete 4-dimensional Lorentz manifold $< R^{(4)}, g^{(4)} >$ (named space-time).

Our purpose is to give an opportunity to take into account presence parallel universes, i.e. other universes being most various 4-dimensional pseudo-Riemannian manifolds which are belonged to special hyperspace of any dimension.

Moreover, hyperspaces should be as is wished much; the geometry, topology, dimension of hyperspaces should be as much as various that always it was possible to find uncountable number of the universes as much as similar to ours, and simultaneously should exist as is wished a lot of installed, completely unlike the world in which we live.

The structure of a physical reality should take into account whim of a conceiving essence to see it in every possible conceivable forms, having thus rather poor research toolkit which basis should be the theory of a relativity and the quantum mechanics.

We are not going to pass to many-dimensional theories such as Kaluza-Klein theory. No. We emphasize that a basis of the Multiverse theory should be the 4-dimensional metric $g^{(4)}$.

1 Formal theory of Multiverse

We create the theory of Multiverse as formal theory $\mathcal{T}$ which is maximally similar to the General theory of Relativity, i.e. as theory of *one* 4-dimensional universe, but other parallel universes must appear under costruction of models of formal theory.

The basis of our formal theory $\mathcal{T}$ is the Kock-Lawvere Synthetic Differential Geometry (SDG) [2].

SDG has not any set-theoretic model because Kock-Lawvere axiom is incompatible with Law of excluded middle. Hence we shall construct formal theory of Multiverse with intuitionistic logic. Models for this theory are topos-theoretic models.

In SDG the commutative ring R is used instead of real field $\mathbb{R}$. The ring R must satisfy the following axioms [1]:

(A1) $< R, +, \cdot, 0, 1 >$ is commutative ring.

(A2) R is local ring, i.e.

$$\begin{array}{l} 0 = 1 \Longrightarrow \perp \\ \exists y\ (x \cdot y = 1) \exists y\ (1 - x) \cdot y = 1. \end{array}$$

(A3) $< R, < >$ is real Euclidean ordered local ring, i.e. $<$ is transitive relation such that

$$\begin{array}{l} (a)\ 0 < 1,\ (0 < x\ \&\ 0 < y \Longrightarrow 0 < x + y\ \&\ 0 < x \cdot y), \\ (b)\ \exists y (x \cdot y = 1) \Longleftrightarrow (0 < x \vee x < 0), \\ (c)\ 0 < x \Longrightarrow \exists y (x = y^2)\ (\textit{Euclidean property}). \end{array}$$

(A4) $\leq$ is a preorder, i.e. reflexive and transitive relation, and

$$\begin{array}{l} (a)\ 0 \leq 1,\ (0 \leq x\ \&\ 0 \leq y \Longrightarrow 0 \leq x + y\ \&\ 0 \leq x \cdot y),\ 0 \leq x^2, \\ (b)\ (x \text{ is nilpotent, i.e. } x^n = 0) \Longrightarrow 0 \leq x. \end{array}$$

(A5) $<$ and $\leq$ are compactible in the following sence:

$$\begin{array}{l} (a)\ x < y \Longrightarrow x \leq y, \\ (b)\ x < y\ \&\ y \leq x \Longrightarrow \perp. \end{array}$$

(A6) (Kock-Lawvere axiom). Let $D = \{x \in R : x^2 = 0\}$. Then

$$\forall (f \in R^D) \exists ! (a, b) \in R \times R\ \forall d \in D (f(d) = a + b \cdot d).$$

(A7) (Integration axiom).

$$\forall f \in R^{[0.1]} \exists ! g \in R^{[0.1]} (g(0) = 0\ \&\ \forall x \in [0, 1]\ (g'(x) = f(x)),$$

where $[0, 1] = \{x \in R : 0 \leq x\ \&\ x \leq 1\}$ and $g'(x)$ is the only b such that $\forall d \in D (g(x + d) = g(x) + b \cdot d)$.

We use the symbolic record:

$$g(x) = \int_0^1 f(t) dt.$$

[1]We give some axioms. Other axioms see in [7, Ch.VII].

(A8) $\forall x \in [0,1]\ (0 < f(x) \Longrightarrow 0 < \int\limits_0^1 f(x)dx)$.

(A8′) $\forall x \in [0,1]\ (0 \leq f(x) \Longrightarrow 0 \leq \int\limits_0^1 f(x)dx)$.

(A9) (Inverse function theorem).

$$\forall f \in R^R \forall x \in R (f'(x) \text{ inversible} \Longrightarrow$$

$$\Longrightarrow \exists \text{ open } U, V (x \in U \ \&\ f(x) \in V \ \&\ f|_U \to V \text{ is a bijection})).$$

(A10) $N \subset R$, i.e. $\forall x \in N\ \exists y \in R(x = y)$.

(A11) R is Archimedean for N, i.e. $\forall x \in R\ \exists n \in N(x < n)$.

(A12) (Peano axioms).

$$\begin{array}{l} 0 \in N \\ \forall x \in R\ (x \in N \Longrightarrow x + 1 \in N) \\ \forall x \in R\ (x \in N \ \&\ x + 1 = 0 \Longrightarrow \perp). \end{array}$$

Ring R includes real numbers from $\mathbb{R}$ and has new elements named *infinitesimals* belonging to "sets"

$$D = \{d \in R : d^2 = 0\}, ..., D_k = \{d \in R : d^{k+1} = 0\}, ...,$$

$$\mathbb{\Delta} = \{x \in R : f(x) = 0, \text{ all } f \in m_0^g\},$$

where $m_{\{0\}}^g$ is ideal of functions having zero germ at 0, i.e. vanishing in a neighbourhood of 0.

We have

$$D \subset D_2 \subset ... \subset D_k \subset ... \subset \mathbb{\Delta}.$$

For given system of axioms we can construct [4, 3] Riemmanian geometry for four-dimensional (formal) manifolds $< R^4, g^{(4)} >$. These manifolds are basis for the Einstein theory of gravitation.

We postulate that *multiverse is four-dimensional space-time in SDG, i.e. is a formal Lorentz manifold* $< R^4, g^{(4)} >$ *for which the Einstein field equations are held:*

$$R_{ik}^{(4)} - \frac{1}{2} g_{ik}^{(4)} (R^{(4)} - 2\Lambda) = \frac{8\pi G}{c^4} T_{ik}. \tag{1}$$

A solution of these equations is 4-metric $g^{(4)}$.

Below we consider the physical consequences of our theory in so called well-adapted models of the form $\mathbf{Set}^{\mathbb{L}^{op}}$ which contain as full subcategory the category of smooth manifolds $\mathcal{M}$.

2 Smooth topos models of multiverse

Let $\mathbb{L}$ be dual category for category of finitely generated C^∞-rings. It is called *category of loci* [7]. The objects of $\mathbb{L}$ are finitely generated C^∞-rings, and morphisms are reversed morphisms of category of finitely generated C^∞-rings.

The object (locus) of $\mathbb{L}$ is denoted as ℓA, where A is a C^∞-ring. Hence, $\mathbb{L}$-morphism $\ell A \to \ell B$ is C^∞-homomorphism $B \to A$.

A finitely generated C^∞-ring ℓA is isomorphic to ring of the form $C^\infty(\mathbb{R}^n)/I$ (for some natural number n and some finitely generated function ideal I).

Category $\mathbf{Set}^{\mathbb{L}^{op}}$ is topos. We consider topos $\mathbf{Set}^{\mathbb{L}^{op}}$ as model of formal theory of multiverse. Only some from axioms (A1)-(A12) are true in topos model $\mathbf{Set}^{\mathbb{L}^{op}}$ [2].

With the Deutsch point of view the transition to concrete model of formal theory is creation of *virtual reality* [3]. Physical Reality that we perceive was called by Deutsch *Multiverse* [4]. Physical Reality is also virtual reality which was created our brain [1, p.140].

A model of multiverse is *generator of virtual reality* which has some *repertoire of environments*. Generator of virtual reality creates environments and we observe them. Explain it.

Under interpretation $i : \mathbf{Set}^{\mathbb{L}^{op}} \models \mathcal{T}$ of formal multiverse theory $\mathcal{T}$ in topos $\mathbf{Set}^{\mathbb{L}^{\mathbf{op}}}$ the objects of theory, for example, ring R, power R^R and so on are interpreted as objects of topos, i.e. functors $F = i(R)$, $F^F = i(R^R)$ and so on. Maps, for example, $R \to R$, $R \to R^R$ are now morphisms of topos $\mathbf{Set}^{\mathbb{L}^{op}}$, i.e. natural transformations of functors: $F \to F$, $F \to F^F$.

Finelly, under interpretation of language of formal multiverse theory we must interpret elements of ring R as "elements" of functors $F \in \mathbf{Set}^{\mathbb{L}^{op}}$. In other words we must give interpretation for relation $r \in R$. It is very difficult

[2] One can take as models topoi $\mathcal{F}, \mathcal{G}$ $\mathcal{Z}$ and others [7, Appendix 2]. All axioms (A1)-(A12) are true for these topoi (see [7, p.300])

[3] This thought belong to Artem Zvaygintsev.

[4] Multiverse = many (multi-) worlds; universe is one (uni) world.

task because functor F is defined on category of loci $\mathbb{L}$; its independent variable is arbitrary locus ℓA, and dependent variable is a set $F(\ell A) \in \mathbf{Set}$. To solve this problem we consider *generalized elements* $x \in_{\ell A} F$ of functor F.

Generalized element $x \in_{\ell A} F$, or *element* x *of functor* F *at stage* ℓA, is called element $x \in F(\ell A)$.

Now we element $r \in R$ interpret as generalized element $i(r) \in_{\ell A} F$. We have such elements so much how much loci. Transition to model $\mathbf{Set}^{\mathbb{L}^{op}}$ causes "reproduction" of element r. It begins to exist in infinite number of variants $\{i(r) : i(r) \in_{\ell A} F, \ell A \in \mathbb{L}\}$.

Note that since 4-metric $g^{(4)}$ is element of object $R^{R^4 \times R^4}$ then "intuitionistic" 4-metric begins to exist in infinite number of variants $i(g)^{(4)} \in_{\ell A} i(R^{R^4 \times R^4})$. Denote such variant as $i(g)^{(4)}(\ell A)$.

For simplification of interpretation we shall operate with objects of models $\mathbf{Set}^{\mathbb{L}^{op}}$. In other words, we shall write $g^{(4)}(\ell A)$ instead of $i(g)^{(4)}(\ell A)$.

Every variant $g^{(4)}(\ell A)$ of 4-metric $g^{(4)}$ satisfies to "own" Einstein equations [4]

$$R^{(4)}_{ik}(\ell A) - \frac{1}{2} g^{(4)}_{ik}(\ell A)[R^{(4)}(\ell A) - 2\Lambda(\ell A)] = \frac{8\pi G}{c^4} T_{ik}(\ell A). \tag{2}$$

(Constants c, G can have different values for different stages ℓA).

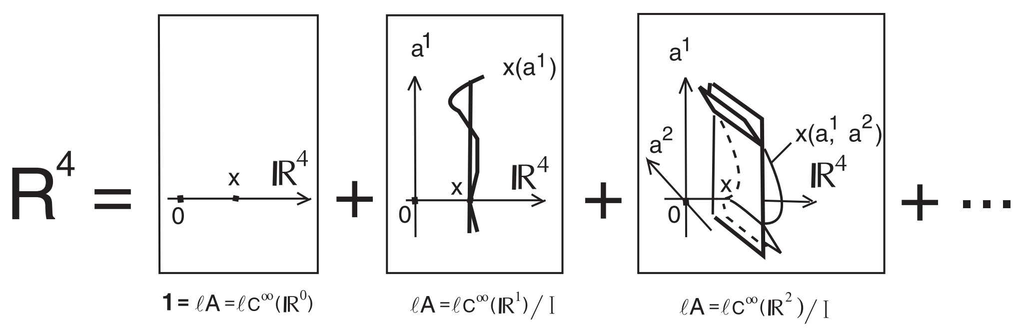

Figure 1: Physical (virtual) Reality R^4 as sum of many-dimensional hyperspaces (environments) $R^4(\ell A)$. Every hyperspace contains a foliation which consists of parallel 4 - dimensional universes ($a = const$).

Previously before going any farther, we shall point to existance of Yoneda embedding

$$y : \mathbb{L} \hookrightarrow \mathbf{Set}^{\mathbb{L}^{op}},$$

$$y(\ell A) = Hom_{\mathbb{L}}(-, \ell A).$$

Assume that ring R is interpreted as functor $y(\ell C^\infty(\mathbb{R}))$, i.e. $i(R) = y(\ell C^\infty(\mathbb{R}))$. Write ℓA instead of $y(\ell A)$ and omit symbol i. Then we have

$$R(-) = \ell C^\infty(\mathbb{R})(-) = Hom_{\mathbb{L}}(-, \ell C^\infty(\mathbb{R})).$$

Similarly

$$R^{R^4\times R^4}(\ell A) = Hom_{\mathbb{L}}(\ell A, R^{R^4\times R^4}) = Hom_{\mathbb{L}}(\ell A \times (R^4 \times R^4), R) =$$

$$= Hom_{\mathbb{L}}(\ell C^\infty(\mathbb{R}^m)/I \times \ell C^\infty(\mathbb{R}^4) \times \ell C^\infty(\mathbb{R}^4), \ell C^\infty(\mathbb{R})) =$$
$$= Hom_{\mathbb{L}^{op}}(\ell C^\infty(\mathbb{R}), C^\infty(\mathbb{R}^m)/I \otimes_\infty C^\infty(\mathbb{R}^4) \otimes_\infty C^\infty(\mathbb{R}^4)) =$$
$$= Hom_{\mathbb{L}^{op}}(C^\infty(\mathbb{R}), C^\infty(\mathbb{R}^{m+8})/(I, \{0\})) =$$
$$= Hom_{\mathbb{L}}(\ell C^\infty(\mathbb{R}^{m+8})/(I, \{0\}), \ell C^\infty(\mathbb{R})),$$

where $\ell A = \ell C^\infty(\mathbb{R}^m)/I$, $\otimes_\infty$ is symbol of coproduction of C^∞-rings and under calculation the folowing formulas are used:

$$C^\infty(\mathbb{R}^n) \otimes_\infty C^\infty(\mathbb{R}^k) = C^\infty(\mathbb{R}^{n+k}),$$

$$\frac{\ell A \to \ell C^{\ell B}}{\ell B \times \ell A \to \ell C}.$$

It follows from this that when $\ell A = \ell C^\infty(\mathbb{R}^m)$ then

$$g^{(4)}(\ell A) = [g \in_{\ell A} R^{R^4\times R^4}] \equiv g^{(4)}_{ik}(x^0, ..., x^3, a)dx^i dx^k, \quad a = (a^1, ..., a^m) \in \mathbb{R}^m.$$

Four-dimensional metric $g^{(4)}_{ik}(x^0, ..., x^3, a)$ we extend to (4+m)-metric in space $\mathbb{R}^{4+m}$

$$g^{(4)}_{ik}(x^0, ..., x^3, a)dx^i dx^k - da^{1^2} - ... - da^{m^2}. \tag{3}$$

We get $(4 + m)$-dimensional geometry.

Symbolically procedure of creation of many-dimensional variants of geometry by means of intuitionistic 4-geometry $g^{(4)}$ one can represent in the form of formal sum

$$g^{(4)} = c_0 \underbrace{[g^{(4)} \in_{\mathbf{1}} R^{R^4\times R^4}]}_{\text{4-geometry}} + c_1 \cdot \underbrace{[g^{(4)} \in_{\ell C^\infty(\mathbb{R}^1)} R^{R^4\times R^4}]}_{\text{5-geometry}} + ...$$

$$... + c_{n-4} \cdot \underbrace{[g^{(4)} \in_{\ell C^\infty(\mathbb{R}^{n-4})} R^{R^4\times R^4}]}_{\text{n-geometry}} + ...,$$

where coefficients c_m are taked from the field of complex numbers.

Because number of stages is infinite, we must write integral instead of sum:

$$g^{(4)} = \int\limits_{\mathbb{L}} \mathcal{D}[\ell A] c(\ell A) [g^{(4)} \in_{\ell C^\infty(\mathbb{R}^{n-4})} R^{R^4 \times R^4}]. \tag{4}$$

Use denotations of quantum mechanics [5]:

$$g^{(4)} \to |g^{(4)}\rangle, \quad [g^{(4)} \in_{\ell C^\infty(\mathbb{R}^{n-4})} R^{R^4 \times R^4}] \to |g^{(4)}(\ell A)\rangle.$$

Then (4) is rewrited in the form

$$|g^{(4)}\rangle = \int\limits_{\mathbb{L}} \mathcal{D}[\ell A] c(\ell A) |g^{(4)}(\ell A)\rangle. \tag{5}$$

Consequently, formal the Kock-Lawvere 4-geometry $< R^4, g^{(4)} >$ is infinite sum of of classical many-dimensional pseudo-Riemmanian geometries which contain foliation of 4-dimensional parallel universes (leaves) (under fixing $a = const$). Geometrical properties of these universes as it was shown in [9, 10] to be different even within the framework of one stage ℓA. About nature of coefficients $c(\ell A)$ we say below in §5.

Now we recall about environments of virtual reality which must appear under referencing to model of multiverse, in this instance, to model $\mathbf{Set}^{\mathbb{L}^{op}}$. This model is generator of virtual reality. It is not difficult to understand that generalised element $|g^{(4)}(\ell A)\rangle$ is metric of concrete environment (=hyperspace $R^4(\ell A)$) with "number" ℓA. In other words, study of any object of theory $\mathcal{T}$ at stage ℓA is transition to one of the environments from repertoire of virtual reality generator $\mathbf{Set}^{\mathbb{L}^{op}}$.

3 The Deutsch-Gödel Multiverse

As example of multiverse we consider cosmological solution of Kurt Gödel [5]

$$g_{ik}^{(4)} = \alpha^2 \begin{pmatrix} 1 & 0 & e^{x^1} & 0 \\ 0 & -1 & 0 & 0 \\ e^{x^1} & 0 & e^{2x^1}/2 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{6}$$

[5] Dirac denotations: $|P\rangle = \psi(\xi)\rangle \equiv \psi(\xi)$; in given case $\psi(\xi)$ is $g^{(4)}$ (representative of state $|P\rangle$), and $|P\rangle$ is $|g^{(4)}\rangle$ [6, p.111-112].

This metric satisfies the Einstein equations (1) with energy-momentum tensor of dust matter

$$T_{ik} = c^2 \rho u_i u_k,$$

if

$$\frac{1}{\alpha^2} = \frac{8\pi G}{c^2}\rho, \quad \Lambda = -\frac{1}{2\alpha^2} = -\frac{4\pi G\rho}{c^2}. \tag{7}$$

Take

$$\alpha = \alpha_0 + d, \ \Lambda = \Lambda_0 + \lambda, \ \rho = \rho_0 + \varrho, \tag{8}$$

where $d, \lambda, \varrho \in D$ are infinitesimals and substitute these in (7). We get

$$\frac{1}{(\alpha_0 + d)^2} = \frac{1}{\alpha_0^2} - \frac{2d}{\alpha_0^3} = \frac{8\pi G}{c^2}(\rho_0 + \varrho),$$

$$2\Lambda_0 + 2\lambda = -\frac{1}{\alpha_0^2} + \frac{2d}{\alpha_0^3}, \quad \Lambda_0 + \lambda = -\frac{4\pi G\rho_0}{c^2} - \frac{4\pi G\varrho}{c^2}.$$

Suppose that $\alpha_0, \Lambda_0, \rho_0 \in \mathbb{R}$ are satisfied to relations (7). Then

$$\lambda = -\frac{4\pi G}{c^2}\varrho, \quad d = -\frac{4\pi G\alpha_0^3}{c^2}\varrho.$$

Under interpretation in smooth topos $\mathbf{Set}^{\mathbb{L}^{op}}$ infinitesimal $\varrho \in D$ at stage $\ell A = C^\infty(\mathbb{R}^m)/I$ is class of smooth functions of the form $\varrho(a) mod\ I$, where $[\varrho(a)]^2 \in I$ [7, p.77].

Consider the properties of the Deutsch-Gödel multiverse at stage $\ell A = \ell C^\infty(\mathbb{R})/(a^4)$ [6], where $a \in \mathbb{R}$. Obviously that it is possible to take infinitesimal of form $\varrho(a) = a^2$. Multiverse at this stage is 5-dimensional hyperspace. This hyperspace contains a foliation, leaves of which are defined by the equation $a = const$. The leaves are parallel universes in hyperspace (environment) $R^4(\ell A)$ with metric $g^{(4)}(\ell A) = g^{(4)}_{ik}(x, a)$ defined formulas (6), (8). Density of dust matter $\rho = \rho_0 + \varrho(a)$ grows from classical value $\rho_0 \sim 2 \cdot 10^{-31}\ g/cm^3$ to $+\infty$ under $a \to \pm\infty$. Cosmological constant grows also infinitely to $-\infty$. Hence parallel universes have different from our Universe physical properties.

At stage $\ell A = \ell C^\infty(\mathbb{R})/(a^2)$ $\varrho(a) = a$ and $\rho = \rho_0 + \varrho(a) \to -\infty$ under $a \to -\infty$, i.e. ρ is not phisically interpreted (we have "exotic" matter with negative density).

Finally, at stage $\mathbf{1} = \ell C^\infty(\mathbb{R})/(a)$ all $\varrho(a) = d(a) = \lambda(a) = 0$, i.e. we have classical the Gödel universe.

[6] As $(f_1, ..., f_k)$ is denoted ideal of ring $C^\infty(\mathbb{R}^n)$ generated dy functions $f_1, ..., f_k \in C^\infty(\mathbb{R}^n)$, i.e. having the form $\sum_{i=1}^k g_i f_i$, where $g_1, ..., g_k \in C^\infty(\mathbb{R}^n)$ are arbitrary smooth functions.

4 Quantum properties of parallel universe geometry

We apply the ideas of the Wheeler quantum geometrodynamics to our formal theory of multiverse. So, formula for probability amplitude of transition from 3-geometry $g^{(3)}$ of physical 3-space to 3-geometry $h^{(3)}$ has the form of "double" Feinman integral over 4-dimensional trajectories $g^{(4)}$:

$$\langle g^{(3)}|h^{(3)}\rangle = \int\limits_{\mathbb{L}} \mathcal{D}[\ell A] \int\limits_{g^{(3)}(\ell A)}^{h^{(3)}(\ell A)} \mathcal{D}[g^{(4)}(\ell A)] e^{\frac{i}{\hbar}S[g^{(4)}(\ell A)]},$$

where

$$S[g^{(4)}(\ell A)] = \kappa_m(\ell A) \int\limits_{\mathbb{R}^{4+m}} \sqrt{-det||g^{(4)}(\ell A)||} R^{(4)}(\ell A) d^4x da^m$$

is action in space $< \mathbb{R}^{4+m}, g^{(4)}(\ell A) >$.

We see that this Feinman integral over trajectories $g^{(4)}$ is infinite number of integrals over $(4+m)$-dimensional trajectories $g^{(4)}(\ell A)$ of the form (3).

We can found quantum fluctuations of 4-metric $g^{(4)} \to g^{(4)} + \Delta g^{(4)}$ which do not give any distortion in interference picture.

Assume that $det||g^{(4)}(\ell A)|| \sim 1$. Then we get for fluctuations in $(4+m)$-dimensional domain with sizes $L^4 \times L_1^m$:

$$\Delta g^{(4)}(\ell A) \sim \frac{L^*}{L}\left(\frac{T}{L_1}\right)^{\frac{m}{2}}, \tag{9}$$

where

$$L^* = \sqrt{\frac{G\hbar}{c^3}} \sim 10^{-33} cm$$

is Planck length. Here $\kappa_m(\ell A) \sim c^3/(\hbar G T^m)$, where T $[cm]$ is value characterizing "size" of additional dimensions.

It follows from (9) that under $L \sim L^*, L_1 \sim T$ all fluctuations $\Delta g^{(4)}(\ell A) \sim 1$, i.e. geometry and topology froth.

As it is shown in [13, 14] fluctuations can take a place at large scale of space and time. Here the main role belongs to additional dimensions which are appeared under consideration of multiverse state at different stages ℓA.

5 Electrons-twins

Deutsch has expected that parallel universe is formed from *shadow* elementary particles accompanying each *real* particle. The real particles we can see or find by means of instruments, but the shadow particles are invisible. They can be found only through their influence with real particles [1, p.48]. "Between real and shadow photons does not exist any differences: each photon is perceived in one universe and is not perceived in all other parallel universes".

The Dirac equation in SDG

$$i\hbar\gamma^{(k)}\frac{\partial\psi}{\partial x^k} - mc\psi = 0, \tag{10}$$

for Minkowsky space-time, i.e. in the Deutsch-Minkowsky multiverse M^4 with metric

$$ds^2 = dx^{0^2} - dx^{1^2} - dx^{2^2} - dx^{3^2}, \tag{11}$$

has, for example, the following solution

$$\psi(x) = \begin{pmatrix} 1 \\ 1 \\ -1 \\ 1 \end{pmatrix} e^{\frac{mc}{\hbar}x^2 + g(x^3+x^0) + i\theta\cdot f(x^3+x^0)}. \tag{12}$$

This solution under $\theta \cdot f(x^3 - x^0) = const$ is spinor ghost [7], i.e. has zero energy-momentum tensor of field $\psi(x)$:

$$\begin{aligned} T_{ik} = \frac{i\hbar c}{4}\Big\{ & \psi^*\gamma^{(0)}\gamma_{(i)}\frac{\partial\psi}{\partial x^k} - \frac{\partial\psi^*}{\partial x^k}\gamma^{(0)}\gamma_{(i)}\psi + \\ & + \psi^*\gamma^{(0)}\gamma_{(k)}\frac{\partial\psi}{\partial x^i} - \frac{\partial\psi^*}{\partial x^i}\gamma^{(0)}\gamma_{(k)}\psi\Big\}. \end{aligned} \tag{13}$$

Hence, spinor ghost ψ does not possess neither energy, nor momentum. So they can not be fixed any instrument. E.V. Palesheva has offerred [15] to identify the spinor ghosts with the Deutsch shadow particles.

Solution ψ is connected [8] with Dirac ket-vector $|\Psi\rangle$ represented in the form of sum [9]

$$|\Psi\rangle = \int\limits_{\mathbb{L}} \mathcal{D}[\ell A] a(\ell A)|\Psi(\ell A)\rangle. \tag{14}$$

---

[7] This solution was found by Elena Palesheva.

[8] See note 5.

[9] The given formula has relation to the Everett interpretation of quantum mechanics [8].

We interpret $\psi = |\Psi\rangle$. Then $\psi^*\psi = \langle\Psi|\Psi\rangle$ is probability amplitude of electron and

$$\int\limits_{R^4} \psi^*\psi d^4x = \int\limits_{R^4} \langle\Psi|\Psi\rangle d^4x = 1. \tag{15}$$

Let

$$\langle\Psi| = \int\limits_{\mathbb{L}} \mathcal{D}[\ell B] a^*(\ell B)\langle\Psi(\ell B)|.$$

So

$$1 = \int\limits_{R^4} \langle\Psi|\Psi\rangle d^4x = \int\limits_{\mathbb{R}^4} d^4x \int\limits_{\mathbb{L}} \mathcal{D}[\ell B] \int\limits_{\mathbb{L}} \mathcal{D}[\ell A] a^*(\ell B) a(\ell A) \langle\Psi(\ell B)|\Psi(\ell A)\rangle =$$

$$= \int\limits_{\mathbb{L}} \mathcal{D}[\ell B] a^*(\ell B) \int\limits_{\mathbb{L}} \mathcal{D}[\ell A] a(\ell A) \left(\int\limits_{\mathbb{R}^4} d^4x \langle\Psi(\ell B)|\Psi(\ell A)\rangle \right) =$$

$$= \int\limits_{\mathbb{L}} \mathcal{D}[\ell B] a^*(\ell B) \int\limits_{\mathbb{L}} \mathcal{D}[\ell A] a(\ell A) \delta(\ell B - \ell A) = \int\limits_{\mathbb{L}} \mathcal{D}[\ell B] a^*(\ell B) a(\ell B),$$

where we take (as logical extension of equality (15)) that

$$\int\limits_{\mathbb{R}^4} d^4x \langle\Psi(\ell B)|\Psi(\ell A)\rangle = \delta(\ell B - \ell A),$$

$$\int\limits_{\mathbb{L}} \mathcal{D}[\ell B] f(\ell B) \delta(\ell B - \ell A) = f(\ell A).$$

Hence

$$\int\limits_{\mathbb{L}} \mathcal{D}[\ell A] a^*(\ell A) a(\ell A) = 1$$

and we can assume that $a^*(\ell A)a(\ell A)$ is probability amplitude of stage ℓA characterizing probability of observation of electron at stage ℓA of multiverse M^4.

Such conclusion one allows to interpret $c^*(\ell A)c(\ell A)$, where $c(\ell A)$ is complex coefficient in decomposition (5) of 4-metric of multiverse $< R^4, g^{(4)} >$, as probability (more exactly, amplitude of probability) that multiverse is inhered in state $|g^{(4)}(\ell A)\rangle$ [10].

Take in (12) number $\theta = 1 - \varepsilon$, where ε infinitesimal, i.e. $\epsilon \in \mathbb{\Delta} = \{x \in R | f(x) = 0, \text{ all } f \in m^g_{\{0\}}\}$, $m^g_{\{0\}}$ is ideal of functions having zero germ at 0.

If $\epsilon \in \mathbb{\Delta}$ then ε at stage $\ell C^\infty(\mathbb{R}^n)/I$ is defined by function $\varepsilon(a), a \in \mathbb{R}^n$ such that for any $\phi \in m^g_{\{0\}} \quad \phi(\varepsilon(a)) \in I$ [7, p.77].

We have

$$\phi(\varepsilon(a)) = \phi(\varepsilon(0)) + \sum_{|\alpha|=1}^{\infty} \frac{1}{\alpha!} D^\alpha(\phi \circ \varepsilon)(0) a^\alpha =$$

$$= \phi(\varepsilon(0)) + \sum_{|\alpha|=1}^{\infty} \frac{1}{\alpha!} \left(\sum_{|\beta|=1}^{|\alpha|} D^\beta \phi(\varepsilon(0)) P_\beta(\varepsilon(0)) \right) a^\alpha, \tag{16}$$

where α, β are multi-indexes and P_β are some polynomials.

At stage $\ell C^\infty(\mathbb{R}^n) \quad \phi(\varepsilon(a)) \in I = \{0\}$ for all $\phi \in m^g_{\{0\}}$. So it follows from (16) that $\phi(\varepsilon(0)) = 0$, and $\varepsilon(0) = 0$. Moreover

$$\sum_{|\beta|=1}^{|\alpha|} D^\beta \phi(\varepsilon(0)) P_\beta(\varepsilon(0)) = 0.$$

But for any $\phi \in m^g_{\{0\}} \quad D^\beta \phi(0) = 0$. Hence $\epsilon(a)$ is arbitrary function satisfing the condition $\varepsilon(0) = 0$.

For field (12) we take that $\theta(a) = 1 - \varepsilon(a)$, where

$$\varepsilon(0) = 0, \ \ \varepsilon(a) > 0 \text{ under } a \neq 0, \text{ and } \epsilon(a) = 1 \text{ under } ||a|| \geq r_0,$$

and f is some non-zero function. Then we have at stage $\ell A = \ell C^\infty(\mathbb{R}^n)$:

$$\theta(a) = 1 - \varepsilon(a) = \begin{cases} 0 \quad \text{under } ||a|| \geq r_0, \\ > 0 \text{ under } ||a|| < r_0. \end{cases}$$

Hence at stage $\ell A = \ell C^\infty(\mathbb{R}^n)$ field ψ is not spinor ghost in our Universe ($a = 0$) and in all universes with $||a|| < r_0$, but is ghost in papallel universes for which $||a|| \geq r_0$. We can take number r_0 so small that universes "labeled"

[10] Metric is gravitational field defining geometry and in some sense topology of space-time. So it is naturally to identify the state (the environment) $|R^4(\ell A)\rangle$ of multiverse at stage ℓA (see, for instance, pic.1) with state $|g^{(4)}(\ell A)\rangle$ of 4-metric $g^{(4)}$.

by parameter a with $||a|| < r_0$ must be considered as one universe due to quantum foam of topologies and geometries (r_0 is "thickness" of universe). This means that field ψ is real particle in our Universe and shadow particle-twin in all other universes.

If we take $\theta \in \mathbb{A}$ such that

$$\theta(a) > 0 \text{ under } ||a - a_0|| < r_0 \text{ and } \theta(a) = 0 \text{ under } ||a|| > r_0,$$

where $a_0 \neq 0$ and $r_0 < ||a_0||$ then field ψ at stage $\ell C^\infty(\mathbb{R}^n)$ is not spinor ghost in the universe $a = a_0$ having "thickness" r_0, and is ghost, i.e. particle-twin in all other universes including our Universe ($a = 0$).

At stage $\mathbf{1} = \ell C^\infty(\mathbb{R}^0) = \ell C^\infty(\mathbb{R})/(a^1)$ $\theta \cdot f(x^3 + x^0) mod\{a^1\} = f(x^3 + x^0)$. It means that we have usual particle carryinging energy and momentum.

6 Photon ghosts and photons-twins

It is known that ftat monochromatic electro-magnetic wave is described by wave equation

$$\frac{1}{c}\frac{\partial \vec{\mathbf{A}}}{\partial t} = \Delta \vec{\mathbf{A}}$$

and has, for example, the following form

$$\vec{\mathbf{A}} = \vec{\mathbf{A}}_{\mathbf{0}} e^{i(\vec{k}\vec{x} - \omega t)}.$$

Electric and magnetic field strengthes of wave are equal to

$$\vec{\mathbf{E}} = i|\vec{k}|\vec{\mathbf{A}}, \quad \vec{\mathbf{H}} = i[\vec{k} \times \vec{\mathbf{A}}]. \tag{17}$$

For energy-momentum tensor of wave we have

$$T^{ij} = \frac{Wc^2}{\omega^2} k^i k^j,$$

where

$$W = \frac{\vec{\mathbf{E}}^2}{4\pi}$$

is energy density of wave.

It follows from these formulas that under substitution $\vec{\mathbf{A}} \to d\vec{\mathbf{A}}$, where $d \in D$, we can get

$$\vec{\mathbf{E}} \to d\vec{\mathbf{E}} \Longrightarrow \vec{\mathbf{E}}(\ell C^\infty(\mathbb{R})/(a^2)) \neq 0 \text{ under } a \neq 0.$$

But $W \to d^2W = 0$. Hence $T_{ik} \equiv 0$, i.e. we have photon ghost in all universes of multiverse. This photon ghost is electro-magnetic wave which is not carrying neither energy, nor momentum in all universes, except universe with $a = 0$, where it does not exist.

Consider now a number $\vartheta \in R$. Let at stage $\ell C^\infty(\mathbb{R})/I$ it is defined by class functions $\vartheta(a) mod\ I$, where

$$\vartheta(a) = e^{-\gamma|a|^2} - 1, \quad \gamma > 0. \tag{18}$$

We get by means of substitution $\vec{\mathbf{A}} \to \vartheta\vec{\mathbf{A}}$ from (17):

$$\vec{\mathbf{E}} = i\vartheta|\vec{k}|\vec{\mathbf{A}}, \quad \vec{\mathbf{H}} = i\vartheta[\vec{k} \times \vec{\mathbf{A}}], \quad \vec{\mathbf{A}} \neq 0.$$

Then

$$\vec{\mathbf{E}}(\ell C^\infty(\mathbb{R})/(\vartheta^2)) \neq 0,$$

but

$$T^{ij} = \frac{Wc^2}{\omega^2} k^i k^j (\ell C^\infty(\mathbb{R})/(\vartheta^2))\ mod\ (\vartheta^2) = 0.$$

In other words at stage (environment) $\ell C^\infty(\mathbb{R})/(\vartheta^2)$ photons-twins which are not carrying neither energy, nor momentum (i.e. being photon ghosts) are observed in all universes.

7 Virtual reality as topos models of formal multiverse

"Set of real numbers" R in $\mathbf{Set}^{\mathbb{L}^{op}}$ has no many accustomed properties of real numbers from $\mathbb{R}$. Hence existence in environments of this virtual reality generator implies unexpected or unaccustomed facts and phenomena. Some such facts were described in giving paper.

Topos $\mathbf{Set}^{\mathbb{L}^{op}}$ is not unique model for formal theory $\mathcal{T}$. Other models, i.e. other virtual reality generators, will demonstrate new properties, new realities. But it is difficult to say which virtual reality is our own Physical Reality.

References

[1] Deutsch, D. *The Fabric of Reality.* Allen Lane. The Penguin Press, 2000.

[2] Kock, A. *Synthetic Differential Geometry.* Cambridge Univ. Press, 1981.

[3] Guts, A.K., Grinkevich, E.B. *Toposes in General Theory of Relativity.* – Los Alamos E-print paper: gr-qc/9610073 (1996). - http://xxx.lanl.gov/abs/gr-qc/9610073

[4] Guts, A.K. *Intuitionistic theory of space-time* // International geometric school-seminar in memory of N.V. Efimov. Abstracts. Abrau-Dyurso. September 27 - October 4, 1996. P.87-88.

[5] Gödel, K. *An Example of a New Type of Cosmological Solution of Einstein's Field Equations of Gravitation.* // Rev. Mod. Phys. 1949. V.21, No.3. P.447-450.

[6] Dirac, P. *Principles of Quantum Mechanics.* Moscow: Nauka, 1979.

[7] Moerdijk, I., Reyes, G.E. *Models for Smooth Infenitesimal Analysis.* Springer-Verlag, 1991.

[8] *Quantum Mechanics of Everett.* – Site in InterNet: http://www.univer.omsk.su/omsk/Sci/Everett.

[9] Guts, A.K., Zvyagintsev, A.A. *Interpretation of intuitionistic solution of the vacuum Einstein equations in smooth topos.* – Los Alamos E-print Paper: gr-qc/0001076 (2000).

[10] Guts, A.K., Zvyagintsev, A.A. *Solution of nearly vacuum Einstein equations in Synthetic Differential Geometry* // Mathematical Structures and Modeling. 2000. No.6. P.115-127.

[11] Guts, A.K., Zvyagintsev, A.A. *Intuitionistic Logic and Signature of Space-time* // Logic and Applications. International Conference on the 60 birthday of Yu.L. Ershov. Abstracts. – Novosibirsk: Institute of Discrete Math. and Informatics. 2000. P.38-39.

[12] Guts, A.K. *Many-valued Logic and multi-variant World* // Logic and Applications. International Conference on the 60 birthday of Yu.L. Ershov. Abstracts. – Novosibirsk: Institute of Discrete Math. and Informatics. 2000. P.36-37.

[13] Guts, A.K. *Interaction of the Past of parallel universes.* - Los Alamos E-print Paper: physics/9910037 (1999).

[14] Guts, A.K. *Models of multi-variant History* // Mathematical Structures and Modeling. 1999. No.4. P.5-14.

[15] Palesheva E.V. *Ghost spinors, shadow electrons and the Deutsch Multiverse.* – Los Alamos E-print paper: gr-qc/0108017 (2001).