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MULTIDIMENSIONAL THEORY OF GRAVITATION AND TIME MACHINES

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This article investigates the possibility of constructing a Time Machine due to a change in the topology of three-dimensional space. The 5-dimensional Kalutsa–Klein theory and the theory of elastic sheets of 4-dimensional foliation in a 5-dimensional manifold are used. Possible energy sources and the time intervals measured by a Time Traveler and by those staying in the initial space section are analyzed.

General relativity allows the possibility for the realization of the idea of Time Machines. It is related to a purely mechanical motion of a body in spacetime along a time loop, i.e., a smooth closed timelike world line. This requires examination of the solutions of the Einstein equations that allow such lines.

We will assume that time loops appear with an artificial change in the topology of spacetime. This is achieved by attaching a 4-handle (4-dimensional wormhole) to spacetime [1, 3]. But the spacetime of general relativity is *absolute*; it does not allow variations and is poorly suited for description of *free* Time Travelers. A richer theory is needed.

Thus, the spacetime model $< W^4, g >$ will be used to describe the Universe as a whole. Our requirement to accomplish travel into the Past means that we switch to the model $< W^4 [f], \hat{g} >$, where $\hat{g}$ is found from g by varying it in the region U surrounding the 4-dimensional handle $[f]$. In the language of physics, the idea is to vary the gravitational field in the "small" region U. The switch from the model $< W^4, g >$ to the model $< W^4 [f], \hat{g} >$ is accomplished by the artificial *free* "growth" of a 4-dimensional handle in proper time with a simultaneous progressive advancement along it to the merger of the "growing" 4-dimensional branch with W^4 at the required place and time already in the spacetime W^4. The dynamics of the "growth" of the 4-handle has been investigated within the framework of 4-dimensional theory in [1, 2]. In essence, all the calculations can be transferred without change to the multidimensional case (see below).

The Traveler moves along the time loop L (in proper time) lying within the 4-handle. The curve L is the world line of the Time Machine. A law is needed to determine the choice of allowed curves L. Such a law cannot be found if one remains in four dimensions. Consequently, one needs to go to multidimensional theories.

We will not be concerned with the construction of the 4-dimensional handle, but assume that it surrounds the curve L from the point of its emergence from W^4 into the 5-th dimension to its re-entry point into W^4. That is, we identify W^4 and $W^4 [f]$ and will not consider in detail the extension of the spacetime branch containing the Time Machine from W^4 to its reemergence with W^4. The construction of the 4-dimensional handle reduces to the problem of constructing the curve L within the scope of 5-dimensional theory. We note that all of this is best combined into a unified picture within the scope of a 6-dimensional theory, but the mathematical difficulties related to foliation theory forces us to be content with 5-dimensional theory.

Our approach corresponds to some degree to the idea of the insignificance of human influence on the geometry of spacetime as a whole, but allows, as will be seen below, the solution of a number of problems with the possible behavior of the curve L with deep penetration into the Past or the Future, i.e., the problems that were bypassed in [1, 2].

Let $< W^4, g >$ be a subspace of the 5-dimensional Lorentz manifold $< V^5, G >$ and $g = G|_W$. It is natural to assume that the curve L has a beginning point a and end point b lying in W^4 and that, in moving along L, we move from a to b with b lying in the Past of a (relative to g). Moreover, one would want the "length" of curve L measured in units of the proper time

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of the Traveler to be very small. We take L to be a timelike curve relative to G. We need to find a geometric construction for which that discussed above is possible. The topology of V^5 in general is unknown, but that of W^4 will be taken equivalent to Euclidean.

1. ELASTIC SPACETIME

Let us assume that $W^4 \subset V^5$ is a sheet of the oriented foliation F of codimensionality 1 in the manifold V^5 [5]. For our purposes, it would be important that the sheet W^4 would behave in the following manner. Let $U_a \subset V^5$ be the vicinity of point a. Then the connectivity component $(W^4 \cap U_a)_a$ of the set $W^4 \cap U_a$ at point a are events close to a in the spacetime W^4. These are events in the Present of the event a with an accuracy to some negligible time interval. If $D \subset W^4$ is the set of events in the remote Past of event a, then we assume that $D \subset U_a \cap W^4$, $D \cap (W^4 \cap U_a)_a = \varnothing$, and there exists a time curve L (relative to the metric G) with beginning a and end $b \in D$. This situation is exactly what is needed for our purposes.

But is such behavior of foliation sheets possible? Perhaps. These are the so-called elastic sheets [4, 5].

The foliation F of codimensionality 1 is specified by the differential 1-form γ, which has the form $\gamma = \gamma_A dx^A$ in the local coordinates x^A ($A = 0, 1, 2, 3, 5$). The form γ must satisfy the Frobenius integrability condition $\gamma \wedge d\gamma = 0$. This means that there exists a 1-form α (defined with accuracy to a factor γ) such that $d\gamma = \alpha \wedge \gamma$. The class of cohomologies of 3-forms $\alpha \wedge d\alpha$ is called the Godbiyon-Vay class of foliation F and is denoted GV(F).

If GV(F) $\neq 0$, then the foliation F has elastic sheets [4, p. 169] which are infinitely interwound. In other words, there are events belonging to the Present in the spacetime W^4, which are arbitrarily close (in the topology of the manifold V^5) in the 5-dimensional space V^5 to events in W^4 of the arbitrarily remote Past (or Future). Motion along the fifth coordinate (in the direction specified by the vector γ^A, dual to the 1-form γ) leads to infinite "skewering" of physical spacetime at points of the Past or Future. The Past is literally next door, one need not look for it in the depths of 5-dimensional space. The metric degree of closeness of the Past is characterized by the vector γ^A and is related to the scalar and electromagnetic fields if one proceeds from the version of 5-dimensional electrogravitational theory discussed in [6].

The second problem is that of specifying the Lorentz metric G on V^5 such that $G|_{W^4} = g$ will also be Lorentz. It can be solved in the following manner.

If the 1-form γ exists, then one can specify locally in the coordinates x^A ($A = 0, 1, 2, 3, 5$) on V^5 the Lorentz metric $\mathring{G}$ of the form (see [6, p. 39], where $\dot{\lambda}$ is written in place of γ)

$$\mathring{G}_{AB} = -\gamma_A \gamma_B + \mathring{g}_{AB},$$
$$\mathring{g}_{5A} = 0,$$

where $\mathring{g}_{AB}$ is the metric tensor of the spacetime W^4.

In 5-dimensional electrogravitational theory with scalar field [6], it is preferable to examine the conformal metric G

$$\begin{gathered} G_{AB} = \varphi^{-2}\mathring{G}_{AB}, \quad g_{AB} = \varphi^{-2}\mathring{g}_{AB}, \quad \varphi = \gamma_5, \\ G_{AB} = -\lambda_A\lambda_B + g_{AB}, \\ \lambda = \varphi^{-1}\gamma, \\ (d\mathring{l})^2 = \mathring{G}_{AB}dx^A dx^B = \varphi^2 G_{AB} dx^A dx^B = \varphi^2 dl^2 \end{gathered} \tag{1}$$

with the additional cylindrical condition denoting that G_{AB} is independent of x^5 as well as the requirement that $G_{55} = -1$. The φ is a scalar field, and the 5-dimensional "Einstein equations" reduce to the 4-dimensional Einstein equations, "Maxwell equations," and Klein–Foch equation for the field φ [6, p. 71]:

$$\begin{aligned} R_{i\kappa}^{(4)} - \frac{1}{2} g_{i\kappa} R^{(4)} - \Lambda\varphi^2 g_{i\kappa} = &-\frac{4G}{c^4}\left(F_{im}F_{\kappa}^{m} - \frac{1}{4} g_{i\kappa} F_{mn} F^{mn}\right) \\ &+ \frac{3}{\varphi}\left(\nabla_i \nabla_\kappa \varphi - g_{i\kappa} g^{mn} \nabla_m \nabla_n \varphi\right) - \frac{6}{\varphi^2}\varphi_{,i}\varphi_{,\kappa} + \kappa Q_{i\kappa}, \end{aligned} \tag{2}$$

$$-\nabla_m F^{m\kappa} - 3F^{m\kappa}\frac{\varphi_{,m}}{\varphi} = \frac{c^2\kappa}{\sqrt{G}}\varphi^3 Q^{\kappa}_{A}\lambda^{A},$$

$$g^{mn}\nabla_m\nabla_n\varphi - \frac{1}{6}R^{(4)}\varphi + \frac{1}{3}\Lambda\varphi^3 - \frac{G\varphi}{2c^4}F_{mn}F^{mn} = -\frac{\kappa}{3}\varphi^{8} Q_{AB}\lambda^{A}\lambda^{B},$$

where $\kappa = 8\pi G/c^4$. The search for elastic spacetime reduces to the calculation of the cohomologous class GV(F) determined in some general 1-form λ directly related to the field φ and 4-potential λ_i (see [5, p. 48]) of the electromagnetic field F_{ik}.

The discussion above obviously has an analog in the 6-dimensional theory of electroweak-gravitational interactions [6] with the metric $G_{AB} = g_{AB} - \lambda_A\lambda_B - \sigma_A\sigma_B$ (A, B = 1, ..., 6), and the existence of elastic spacetime can be investigated with the 1-forms λ and σ [5] related to the neutral vector fields [6].

We note that when the metric G has the signature $< + - - \ldots - >$, penetration into the immediate past is obstructed, since the 5-dimensional light cones cannot slope rapidly because of their continuous dependence on the coordinates of the vertices. There is no such obstacle with the signature choice $< + - - \ldots - + >$, but, as is well known [6, p. 44], this gives the wrong sign in front of the electromagnetic field tensor in Eqs. (2).

2. RECKONING OF TIME ON THE JOURNEY INTO THE PAST

How are the passages of time on the journey into the Past according to the clocks of W^4 and the proper clocks of the Traveler related?

Let us define the differential of the proper time along the timelike curve L as

$$d\tau = \frac{dl}{c}, \quad dl^2 = G_{AB}dx^A dx^B,$$

where the coordinates x^A (A = 0, 1, 2, 3, 5) are specified in the region $U_a \subset V^5$. Let us assume that the Traveler into the Past moves along L in U_a such that he is at rest in $W^4 \cap U_a$, i.e., x^1, x^2, x^3 = const. We then have

$$dl^2 = ds^2 - d\lambda^2, \quad ds^2 = c^2dt^2 - dl^2,$$

where

$$dt^2 = \frac{g_{0i}dx^i}{c\sqrt{g_{00}}}, \quad dl^2 = \left(-g_{\alpha\beta} + \frac{g_{0\alpha}g_{0\beta}}{g_{00}}\right)dx^\alpha dx^\beta \ (\alpha, \beta = 1, 2, 3)$$

are chronometrically invariant with respect to time and length in the spacetime W^4. Then

$$d\tau = \sqrt{1 - \left(\frac{d\lambda}{ds}\right)^2}\frac{ds}{c} = \sqrt{1 - \left(\frac{d\lambda}{ds}\right)^2}\sqrt{1 - \left(\frac{dl}{cdt}\right)^2}dt = \tag{3}$$
$$= \sqrt{1 - \left(\frac{d\lambda}{ds}\right)^2}dt,$$

since dl = 0. However, it is shown in [6, p. 51] that

$$\frac{d\lambda}{ds} = -\frac{e}{2m\sqrt{G}}, \tag{4}$$

where e and m are the electric charge and mass of the Time Machine.

The condition that the vector ξ is tangent to W^4 has the form $d\lambda(\xi) = \lambda_A\xi^A = 0$. This means that motion transverse to W^4 along the curve L: $x^A = x^A(s)$ is characterized by the inequality

$$d\lambda\left(\frac{dx^A}{ds}\right) = \lambda_A\frac{dx^A}{ds} = \frac{d\lambda}{ds} \neq 0. \tag{5}$$

We note in comparing (4) and (5) that transverse motion, i.e., motion into the fifth dimension, requires that the body have an electric charge. Consequently, to launch a Time Machine, it must be given an electric charge. It is then completely admissible in 5-dimensional theory that the Time Machine as a charged test body move along a geodesic in $< V^5, G>$ (see [6, pp. 43, 51]). This means that the 5-dimensional equations of (time) geodesics determine the laws of motion of the Time Machine in V^5.

Equation (3) shows that the condition $(d\lambda/ds)^2 \leq 1$, i.e., $e/2m\sqrt{G} \leq 1$, must be satisfied in order that the time τ not be imaginary. The electron, for example, does not satisfy this restriction. Thus, the initial mass and charge of the Time Machine cannot be arbitrary.

It was shown in [6, p. 78] that

$$\left(\frac{d\lambda}{ds}\right)^2 = \frac{W_0}{\Phi^2 + W_0} = \frac{e^2}{4Gm^2}. \qquad (6)$$

We thus have for the depth of penetration into the fifth dimension with $dl = 0$

$$d\lambda = c\sqrt{\frac{W_0}{\Phi^2 + W_0}}\,dt = c\,\frac{e}{2m\sqrt{G}}\,dt.$$

Consequently, an increase in the scalar field φ or mass of the Time Machine or a decrease in its electric charge reduces the depth of penetration into the fifth dimension and thereby facilitates reaching the part of the elastic W^4 that is the increasingly remote Past or Future. Simultaneously, as the quantity (6) decreases, the proper time of the Traveler into the Past $d\tau$, as is seen from (3), increases toward the time dt fixed in W^4.

3. SEARCH FOR THE ENERGY SOURCE FOR A TIME MACHINE

The energy required to form the 4-handle and, consequently, to launch the Time Machine along the curve L from point a to point b was evaluated in [1, 2]. Such a 4-handle is formed with the "cleavage" of a sphere (containing the Time Machine) of radius l from 3-space due to the change in topology of 3-space. It is this change in topology that allows the "escape" from spacetime W^4 at point a into the fifth dimension into V^5 in order to "return" to W^4 at point b.

To find an energy source with which to operate a Time Machine, let us make use of the 5-dimensional electrogravitational theory based on the metric (1) and Eqs. (2). It follows from the 00-equation of (2) that

$$R^{(3)} + K_2 = \frac{4G}{c^4}\varepsilon_F(t) + 2\varepsilon_\varphi(t) + \frac{16\pi G}{c^4}\varepsilon_Q(t), \qquad (7)$$

where $R^{(3)}$ is the scalar and K_2 is the external (relative to W^4) curvature of 3-space and $\varepsilon_F(t)$, $\varepsilon_\varphi(t)$, and $\varepsilon_Q(t)$ are the energy densities of the electromagnetic field, scalar field, and remaining matter, respectively. Consequently, the mean curvature discontinuity $<\delta R^{(3)}> \sim 2$ responsible for the change in topology of 3-space [1, 2] results from the discontinuities in the energy densities of the electromagnetic field, scalar field, and remaining matter separately or all together. It is seen from (7) that the mean discontinuities of the electromagnetic field and remaining matter must be tremendous [1, 2]:

$$<\delta\varepsilon_F> \sim \frac{c^4}{G}\frac{1}{l^2}, \quad <\delta\varepsilon_Q> \sim \frac{c^4}{4\pi G}\frac{1}{l^2},$$

whereas $<\delta\varepsilon_\varphi> \sim 1$. It then follows from this and the preceding paragraph that the depth of penetration into the Past as well as the rate of passage of the proper time of the Traveler with motion of the Time Machine depend little on the field φ. The equality $\varphi = 1$ characterizes the absence of the field φ. Thus, stepwise oscillations of the field φ comparable to the field itself may lead to a change in the topology of 3-space and to the transfer of matter in time.

4. CONCLUSION

The Time Machine theory discussed above is based on the assumption of the absolute nature of 4-dimensional physical spacetime W^4. By this, we mean the following. All that has existed, is existing, or will exist in the Universe comprises the

World of events called spacetime, which is a unique Something occurring in invariable, eternal material realty. Nothing disappears, everything is fixed. The Past and the Future are just as real as the Present! In that case, the Past is accessible in addition to the natural flow of time through a 4-handle. The hypothesis of the existence of elastic sheets makes this idea feasible.

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