The theory of Multiverse, multiplicity of physical objects and physical constants

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ABSTRACT

The Multiverse is collection of parallel universes. In this article a formal theory and a topos-theoretic models of the multiverse are given. For this the Lawvere-Kock Synthetic Differential Geometry and topos models for smooth infinitesimal analysis are used. Physical properties of multi-variant and many-dimensional parallel universes are discussed. The source of multiplicity of physical objects is set of physical constants.

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1 Intoduction

In the Deutsch 's book [1] the sketch of structure of physical reality named Multiverse which is set of the parallel universes is given. Correct description of the Multiverse can be done only within the framework of the quantum theory.

In this article a sketch of formal theory and topos-theoretic models of the Deutsch multiverse are given (see more in [2]).

We wish to preserve the framework of the mathematical apparatus of the 4-dimensional General theory of Relativity, and so we shall consider the Universe as concrete 4-dimensional Lorentz manifold $< \mathcal{R}^4, g^{(4)} >$ (named space-time).

2 Formal theory of Multiverse

We construct the theory of Multiverse as formal theory $\mathcal{T}$ which is maximally similar to the General theory of Relativity, i.e. as theory of *one* 4-dimensional universe, but other parallel universes must appear under costruction of models of formal theory.

The basis of our formal theory $\mathcal{T}$ is the Kock-Lawvere Synthetic Differential Geometry (SDG) [3].

It is important to say that SDG has no any set-theoretic model because Lawvere-Kock axiom is incompatible with Law of excluded middle. Hence we shall construct formal theory of Multiverse on base of the intuitionistic logic. Models for this theory are smooth topos-theoretic models and for stheir description the usual classical logic is used.

In SDG the commutative ring $\mathcal{R}$ is used instead of real field $\mathbb{R}$. The ring $\mathcal{R}$ must satisfy the following

Lawvere-Kock axiom. *Let* $D = \{x \in \mathcal{R} : x^2 = 0\}$. *Then*

$$\forall (f \in \mathcal{R}^D) \exists ! (a, b) \in \mathcal{R} \times \mathcal{R} \ \forall d \in D (f(d) = a + b \cdot d).$$

and some other axioms (see in [4, Ch.VII].).

Ring $\mathcal{R}$ includes real numbers from $\mathbb{R}$ and has new elements named *infinitesimals* belonging to "sets"

$$D = \{d \in \mathcal{R} : d^2 = 0\}, ..., D_k = \{d \in \mathcal{R} : d^{k+1} = 0\}, ...$$

$$\Delta = \{x \in \mathcal{R} : f(x) = 0, \text{ all } f \in m^g_{\{0\}}\},$$

where $m^g_{\{0\}}$ is ideal of smooth functions having zero germ at 0, i.e. vanishing in a neighbourhood of 0.

We have

$$D \subset D_2 \subset ... \subset D_k \subset ... \subset \mathbb{\Delta}.$$

We can construct Riemmanian geometry for four-dimensional (formal) manifolds $< \mathcal{R}^4, g^{(4)} >$. These manifolds are basis for the Einstein theory of gravitation [5].

We postulate that *multiverse is four-dimensional space-time in SDG, i.e. is a formal Lorentz manifold* $< \mathcal{R}^4, g^{(4)} >$ *for which the Einstein field equations are held:*

$$R^{(4)}_{ik} - \frac{1}{2} g^{(4)}_{ik} (R^{(4)} - 2\Lambda) = \frac{8\pi G}{c^4} T_{ik}. \tag{1}$$

A solution of these equations is 4-metric $g^{(4)}(x)$, $x \in \mathcal{R}$.

Below we consider the physical consequences of our theory in so called well-adapted smooth topos models of the form $\mathbf{Set}^{\mathbb{L}^{op}}$ which contain as full subcategory the category of smooth manifolds $\mathcal{M}$.

3 Smooth topos models of multiverse

Let $\mathbb{L}$ be dual category for category of finitely generated C^∞-rings. It is called *category of loci* [4]. The objects of $\mathbb{L}$ are finitely generated C^∞-rings, and morphisms are reversed morphisms of category of finitely generated C^∞-rings.

The object (locus) of $\mathbb{L}$ is denoted as ℓA, where A is a C^∞-ring. Hence, $\mathbb{L}$-morphism $\ell A \to \ell B$ is C^∞-homomorphism $B \to A$.

A finitely generated C^∞-ring ℓA is isomorphic to ring of the form $C^\infty(\mathbb{R}^n)/I$ (for some natural number n and some ideal I of finitely generated functions).

Category $\mathbf{Set}^{\mathbb{L}^{op}}$ is topos [4]. We consider topos $\mathbf{Set}^{\mathbb{L}^{op}}$ as model of formal theory of multiverse.

With the Deutsch point of view the transition to concrete model of formal theory is creation of *virtual reality* [1]. Physical Reality that we perceive was called by Deutsch *Multiverse* [2]. Physical Reality is also virtual reality which was created our brain [1, p.140].

[1]This thought belongs to Artem Zvyagintsev.

[2]Multiverse = many (multi-) worlds; universe is one (uni) world.

A model of multiverse is *generator of virtual reality* which has some *repertoire of environments*. Generator of virtual reality creates environments and we observe them. Explain it.

Under interpretation $i : \mathbf{Set}^{\mathbb{L}^{op}} \models \mathcal{T}$ of formal multiverse theory $\mathcal{T}$ in topos $\mathbf{Set}^{\mathbb{L}^{\mathbf{op}}}$ the objects of theory, for example, ring $\mathcal{R}$, power $\mathcal{R}^{\mathcal{R}}$ and so on are interpreted as objects of topos, i.e. functors $F = i(\mathcal{R})$, $F^F = i(\mathcal{R}^{\mathcal{R}})$ and so on. Maps, for example, $\mathcal{R} \to \mathcal{R}$, $\mathcal{R} \to \mathcal{R}^{\mathcal{R}}$ are now morphisms of topos $\mathbf{Set}^{\mathbb{L}^{op}}$, i.e. natural transformations of functors: $F \to F$, $F \to F^F$.

Finelly, under interpretation of language of formal multiverse theory we must interpret elements of ring $\mathcal{R}$ as "elements" of functors $F \in \mathbf{Set}^{\mathbb{L}^{op}}$. In other words we must give interpretation for relation $r \in \mathcal{R}$. It is very difficult problem because functor F is defined on category of loci $\mathbb{L}$; its independent variable is arbitrary locus ℓA, and dependent variable is a set $F(\ell A) \in \mathbf{Set}$. To solve this problem we consider *generalized elements* $x \in_{\ell A} F$ of functor F.

Generalized element $x \in_{\ell A} F$, or *element x of functor F at stage ℓA*, is called an element $x \in F(\ell A)$.

Now we element $r \in \mathcal{R}$ interpret as generalized element $i(r) \in_{\ell A} F$, where $F = i(\mathcal{R})$. We have such elements so much how much loci. Transition to model $\mathbf{Set}^{\mathbb{L}^{op}}$ causes "reproduction" of element r. It begins to exist in infinite number of variants $\{i(r) : i(r) \in_{\ell A} F, \ell A \in \mathbb{L}\}$.

Note that since 4-metric $g^{(4)}$ is element of object $\mathcal{R}^{\mathcal{R}^4 \times \mathcal{R}^4}$ then "intuitionistic" 4-metric begins to exist in infinite number of variants $i(g)^{(4)} \in_{\ell A} i(\mathcal{R}^{\mathcal{R}^4 \times \mathcal{R}^4})$. Denote such variant as $i(g)^{(4)}(\ell A)$.

For simplification of interpretation we shall operate with objects of models $\mathbf{Set}^{\mathbb{L}^{op}}$. In other words, we shall write $g^{(4)}(\ell A)$ instead of $i(g)^{(4)}(\ell A)$.

Every variant $g^{(4)}(\ell A)$ of 4-metric $g^{(4)}$ satisfies to "own" Einstein equations [5]

$$R_{ik}^{(4)}(\ell A) - \frac{1}{2} g_{ik}^{(4)}(\ell A)[R^{(4)}(\ell A) - 2\Lambda(\ell A)] =$$

$$= \frac{8\pi G}{c^4} T_{ik}(\ell A).$$

(Constants c, G can also have different values at different stages ℓA).

It follows from theory that when $\ell A = \ell C^{\infty}(\mathbb{R}^m)$ then

$$g^{(4)}(\ell A) = [g \in_{\ell A} \mathcal{R}^{\mathcal{R}^4 \times \mathcal{R}^4}] \equiv g_{ik}^{(4)}(x^0, ..., x^3, a) dx^i dx^k,$$

$$a = (a^1, ..., a^m) \in \mathbb{R}^m.$$

Four-dimensional metric $g^{(4)}_{ik}(x^0, ..., x^3, a)$ we extend to (4+m)-metric in space $\mathbb{R}^{4+m}$

$$g^{(4+m)}_{AB} dx^A dx^B \equiv$$

$$\equiv g^{(4)}_{ik}(x^0, ..., x^3, a) dx^i dx^k - da^{1^2} - ... - da^{m^2}, \tag{2}$$

We get (4+m)-dimensional pseudo-Riemannian geometry $< \mathbb{R}^{4+m}, g^{(4+m)}_{AB} >$.

Symbolically procedure of creation of many-dimensional variants of space-time geometry by means of intuitionistic 4-geometry $< \mathcal{R}^4, g^{(4)} >$ one can represent in the form of formal sum

$$g^{(4)} = c_0 \cdot \underbrace{[g^{(4)} \in_{\mathbf{1}} \mathcal{R}^{\mathcal{R}^4 \times \mathcal{R}^4}]}_{\text{4-geometry}} +$$

$$+ c_1 \cdot \underbrace{[g^{(4)} \in_{\ell C^\infty(\mathbb{R}^1)} \mathcal{R}^{\mathcal{R}^4 \times \mathcal{R}^4}]}_{\text{5-geometry}} + ...$$

$$... + c_{n-4} \cdot \underbrace{[g^{(4)} \in_{\ell C^\infty(\mathbb{R}^{n-4})} \mathcal{R}^{\mathcal{R}^4 \times \mathcal{R}^4}]}_{\text{n-geometry}} + ...,$$

where coefficients c_m are taked from the field of complex numbers.

Because number of stages is infinite, we must write integral instead of sum:

$$g^{(4)} = \int\limits_{\mathbb{L}} \mathcal{D}[\ell A] c(\ell A) [g^{(4)} \in_{\ell C^\infty(\mathbb{R}^{n-4})} \mathcal{R}^{\mathcal{R}^4 \times \mathcal{R}^4}]. \tag{3}$$

Use denotations of quantum mechanics [3]:

$$g^{(4)} \to |g^{(4)}\rangle, \ [g^{(4)} \in_{\ell C^\infty(\mathbb{R}^{n-4})} \mathcal{R}^{\mathcal{R}^4 \times \mathcal{R}^4}] \to |g^{(4)}(\ell A)\rangle.$$

Then (3) is rewrited in the form

$$|g^{(4)}\rangle = \int\limits_{\mathbb{L}} \mathcal{D}[\ell A] c(\ell A) |g^{(4)}(\ell A)\rangle. \tag{4}$$

[3]Dirac denotations: $|P\rangle = \psi(\xi)\rangle \equiv \psi(\xi)$; in given case $\psi(\xi)$ is $g^{(4)}$ (representative of state $|P\rangle$), and $|P\rangle$ is $|g^{(4)}\rangle$ [6, p.111-112].

Consequently, formal the Lawvere-Kock 4-geometry $< \mathcal{R}^4, g^{(4)} >$ is infinite sum

$$\mathcal{R}^4 = \int\limits_{\mathbb{L}} \mathcal{D}[\ell A] c(\ell A) \mathcal{R}^4_{\ell A}$$

of classical many-dimensional pseudo-Riemmanian geometries $\mathcal{R}^4_{\ell A} = =< \mathbb{R}^{4+m}, g^{(4+m)}_{AB}(x,a) >$ every of which contains the foliation of 4-dimensional parallel universes (leaves) (under fixing $a = const$). Geometrical properties of these universes as it was shown in [7, 8] to be different even within the framework of one stage ℓA.

Now we recall about environments of virtual reality which must appear under referencing to model of multiverse, in this instance, to model $\mathbf{Set}^{\mathbb{L}^{op}}$. This model is generator of virtual reality. It is not difficult to understand that generalised element $|g^{(4)}(\ell A)\rangle$ is metric of concrete environment (=hyperspace $\mathcal{R}^4_{\ell A}$) with "number" ℓA. In other words, study of any object of theory $\mathcal{T}$ at stage ℓA is transition to one of the environments from repertoire of virtual reality generator $\mathbf{Set}^{\mathbb{L}^{op}}$.

4 The Gödel-Deutsch Multiverse

As example of multiverse we consider cosmological solution of Kurt Gödel [9]

$$g^{(4)}_{ik} = \alpha^2 \begin{pmatrix} 1 & 0 & e^{x^1} & 0 \\ 0 & -1 & 0 & 0 \\ e^{x^1} & 0 & e^{2x^1}/2 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{5}$$

This metric satisfies the Einstein equations (1) with energy-momentum tensor of dust matter

$$T_{ik} = c^2 \rho u_i u_k,$$

if

$$\frac{1}{\alpha^2} = \frac{8\pi G}{c^2}\rho, \quad \Lambda = -\frac{1}{2\alpha^2} = -\frac{4\pi G\rho}{c^2}. \tag{6}$$

Take

$$\alpha = \alpha_0 + d, \ \Lambda = \Lambda_0 + \lambda, \ \rho = \rho_0 + \varrho, \tag{7}$$

where $d, \lambda, \varrho \in D$ are infinitesimals and substitute these in (6). We get

$$\frac{1}{(\alpha_0 + d)^2} = \frac{1}{\alpha_0^2} - \frac{2d}{\alpha_0^3} = \frac{8\pi G}{c^2}(\rho_0 + \varrho),$$

$$2\Lambda_0 + 2\lambda = -\frac{1}{\alpha_0^2} + \frac{2d}{\alpha_0^3}, \quad \Lambda_0 + \lambda = -\frac{4\pi G\rho_0}{c^2} - \frac{4\pi G\varrho}{c^2}.$$

Suppose that $\alpha_0, \Lambda_0, \rho_0 \in \mathbb{R}$ are satisfied to relations (6). Then

$$\lambda = -\frac{4\pi G}{c^2}\varrho, \quad d = -\frac{4\pi G\alpha_0^3}{c^2}\varrho.$$

Under interpretation in smooth topos $\mathbf{Set}^{\mathbb{L}^{op}}$ infinitesimal $\varrho \in D$ at stage $\ell A = C^\infty(\mathbb{R}^m)/I$ is class of smooth functions of the form $\varrho(a)\ mod\ I$, where $[\varrho(a)]^2 \in I$ [4, p.77].

Consider the properties of the Gödel-Deutsch multiverse at stage $\ell A = \ell C^\infty(\mathbb{R})/(a^4)$ [4], where $a \in \mathbb{R}$. Obviously that it is possible to take infinitesimal of form $\varrho(a) = a^2$. Multiverse at this stage is 5-dimensional hyperspace. This hyperspace contains a foliation, leaves of which are defined by the equation $a = const$. The leaves are parallel universes in hyperspace (environment) $\mathcal{R}^4_{\ell A}$ with metric $g^{(4)}(\ell A) = g^{(4)}_{ik}(x,a)$ defined formulas (5), (7). Density of dust matter $\rho = \rho_0 + \varrho(a)$ grows from classical value $\rho_0 \sim 2 \cdot 10^{-31}\ g/cm^3$ to $+\infty$ under $a \to \pm\infty$. Cosmological constant grows also infinitely to $-\infty$. Hence parallel universes have different from our Universe physical properties.

At stage $\ell A = \ell C^\infty(\mathbb{R})/(a^2)$ $\varrho(a) = a$ and $\rho = \rho_0 + \varrho(a) \to -\infty$ under $a \to -\infty$, i.e. ρ is not phisically interpreted (we have "exotic" matter with negative density).

Finally, at stage $\mathbf{1} = \ell C^\infty(\mathbb{R})/(a)$ all $\varrho(a) = d(a) = \lambda(a) = 0$, i.e. we have classical the Gödel universe.

5 The Friedman-Deutsch Multiverse

Now we consider closed Friedman model of Universe, which in coordinates $(x^0, \chi, \theta, \varphi)$, $x^0 = ct$, has the following metric

$$ds^2 = g^{(4)}_{ik} dx^i dx^k =$$

$$= c^2dt^2 - R^2(t)[d\chi^2 + \sin^2\chi\ (d\theta^2 + \sin^2\theta d\varphi^2)]. \tag{8}$$

[4]Here $(f_1, ..., f_k)$ is ideal of ring $C^\infty(\mathbb{R}^n)$ generated dy functions $f_1, ..., f_k \in C^\infty(\mathbb{R}^n)$, i.e. having the form $\sum_{i=1}^k g_i f_i$, where $g_1, ..., g_k \in C^\infty(\mathbb{R}^n)$ are arbitrary smooth functions.

This metric satisfies the Einstein equations with energy-momentum tensor of dust matter

$$T_{ik} = c^2 \rho u_i u_k,$$

under the condition that

$$\rho R^3(t) = const = \frac{M}{2\pi^2} \tag{9}$$

$$\begin{cases} R = R_0(1 - \cos\eta), \\ t = \frac{R_0}{c}(\zeta - \sin\eta), \end{cases} \tag{10}$$

$$R_0 = \frac{2GM}{3\pi c^3}, \tag{11}$$

where M is sum of body mass in 3-space [10, p.438].

Let

$$G = k + d, \quad d \in D \tag{12}$$

where $k = 6,67 \cdot 10^{-8}$ $[CGS]$ is classical gravitational constant.

At stage $\mathbf{1} = \ell C^\infty(\mathbb{R})/(a)$ $\quad d(a) = 0$, i.e. we have classical Friedman Universe.

Consider the state of the Friedman-Deutsch multiverse at stage $\ell A = \ell C^\infty(\mathbb{R})/(a^4)$, where $a \in \mathbb{R}$. Obviously that it is possible to take infinitesimal of form $\varrho(a) = a^2$. Multiverse at this stage is 5-dimensional hyperspace. This hyperspace contains a foliation, leaves of which are defined by the equation $a = const$. The leaves are parallel universes in hyperspace (environment) $R^4_{\ell A}$ with metric $g^{(4)}(\ell A) = g^{(4)}_{ik}(x, a)$ defined formulas (8)-(11).

Radius of "Universe" with number $a = const$ and dust density as it follows from (9) are equal to

$$R = \frac{2}{3\pi c^3}(k + a^2)(1 - \cos\eta),$$

$$\rho(a) = \frac{27\pi c^3}{16k^3 M^2 (1 - \cos\eta)^3}\left(1 - \frac{3}{k} d(a)\right).$$

So under $d = a^2$ the radius of parallel universes with numbers $|a| \to +\infty$ grows to $+\infty$. The dust density $\rho(a)$ will decreas, then $\rho(a)$ is crossing zero and becomes negative, $\rho(a) \to -\infty$ under $|a| \to +\infty$. All this says that parallel universes can have a physical characteristics which are absolutely different from characteristics of our Universe.

6 Transitions between parallel hyperspaces

Change of stage ℓA on stage ℓB is morphism between two stages

$$\ell B \stackrel{\Phi}{\to} \ell A.$$

When $\ell A = \ell C^\infty(\mathbb{R}^n)$ and $\ell B = \ell C^\infty(\mathbb{R}^m)$. Then transition Φ between stages gives smooth mapping

$$\phi : \mathbb{R}^m \ni b \to a \in \mathbb{R}^n,$$

$$a = \phi(b).$$

Hence if constants $G = G(a), \Lambda = \Lambda(a)$ at stage ℓA, then we have at new stage $\ell B \quad G = G(\phi(b)), \Lambda = \Lambda(\phi(b))$. In other words, dependence of physical constants on exra-dimensions is transformated in dependence of physical constants on some exra-field ϕ. This fact can be useful in connection with investigations, concerning introduction effective fravitational constant depending on some scalar field (see, for example [11]).

7 Conclusion

As it follows from sections 4,5 the source of multiplicity of objects and appearance of parallel hyperspaces are the physical constants (for example, ρ, Λ, G). Reason this in following. Traditionally we consider physical constants as real numbers. It means impossibility of findings of their exact values. So we musr take that physical constant $K = K_0 + d$, where d is an infinitesimal. The last gives multiplicity.

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