Antigravitation in higher dimensional General Relativity

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Let's consider 5-metrics

$$dS^2 = \left[1+\frac{1}{6}(\varkappa c^2\rho_2 a - 2\Lambda_1)ar^2\right]dx^{0 2} - \left[1-\frac{(\varkappa c^2\rho_2 a+\Lambda_1)}{3}\cdot r^2 a\right]^{-1}dr^2 - r^2 d\Omega^2 - da^2,$$

$$d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2,$$

where $\varkappa, \rho_2, \Lambda_1 = const$. Gravitational force, operating on a trial body, in 4-dimensional space-time $< M_a^4, ds^2 > = < (x^0, r, \theta, \varphi), dS^2|_{a=const} >$, is possible to calculate on a formula from [Landau L., Lifshits E. Theory of field. Moscow, 1973. P.327]:

$$f_\alpha = \frac{mc^2}{\sqrt{1-\dfrac{v^2}{c^2}}}\left\{-\frac{\partial}{\partial x^\alpha}\ln\sqrt{g_{00}} + \sqrt{g_{00}}\left[\frac{\partial}{\partial x^\beta}\left(\frac{g_{0\alpha}}{g_{00}}\right) - \frac{\partial}{\partial x^\alpha}\left(\frac{g_{0\beta}}{g_{00}}\right)\right]\frac{v^\beta}{c}\right\}.$$

We have

$$f_r = -\frac{mc^2}{6\sqrt{1-v^2/c^2}}\frac{\left[\varkappa c^2\rho_2 a - 2\Lambda_1\right]ar}{\left[1+\dfrac{1}{6}(\varkappa c^2\rho_2 a - 2\Lambda_1)ar^2\right]},\ \ f_\varphi = f_\theta = 0.$$

In this case, it is obvious that it is possible to find the functions $\Lambda = \Lambda_1 a, \rho = \rho_2 a^2$ so that $\rho_2 > 0$, and f_r changes a sign in $a = 0$ and $a = 2\Lambda_1/(\varkappa c^2\rho_2)$ in extensive spatial area with radius $r < c\sqrt{6\varkappa\rho_2}/|\Lambda_1|$ (Inequality is received as a condition of positivity of a denominator in a formula for f_r for every a.), i.e. the attraction to the center of $r = 0$ is replaced by the repulsion from center $r = 0$.

Transition through $a = 0$ changes a sign of "the cosmological constant" Λ, and observable change of gravitation on antigravitation can be regarded as manifestation of the cosmological repulsion. But upon transition through $a = 2\Lambda_1/(\varkappa c^2\rho_2)$ "the cosmological constant" keeps a sign, and it means that we have other type of antigravitation.

If $r > c\sqrt{6\varkappa\rho_2}/|\Lambda_1|$, then denominator of f_r is remained positive under

$$a > a_+(r) = \frac{1}{\varkappa c^2\rho_2}[\Lambda_1 + \sqrt{\Lambda_1^2 - (6\varkappa c^2\rho_2/r^2)}] \text{ or } a < a_-(r) = \frac{1}{\varkappa c^2\rho_2}[\Lambda_1 - \sqrt{\Lambda_1^2 - (6\varkappa c^2\rho_2/r^2)}].$$

If $\Lambda_1 > 0$, then we have $a_+(r) < 2\Lambda_1/(\varkappa c^2\rho_2)$. Hence, for every $r > c\sqrt{6\varkappa\rho_2}/\Lambda_1$ when the parameter a is changed in some small interval $(2\Lambda_1/(\varkappa c^2\rho_2) - \varepsilon(r), 2\Lambda_1/(\varkappa c^2\rho_2) + \varepsilon(r))$, the function f_r changes a sign, i.e. the attraction is replaced by the repulsion. Under $\Lambda_1 < 0$ we have $2\Lambda_1/(\varkappa c^2\rho_2) < a_-(r)$. Hence, for every $r > c\sqrt{6\varkappa\rho_2}/\Lambda_1$ gravitatiomal force f_r changes sign, when the parameter a is changed in the same interval.

Thus, when we are moving in 5-dimensional bulk, i.e. when a is changed, the geometry of 4-brane M_a^4 is changed so that gravitation (attraction) is replaced with antigravitation (repulsion).