which is differentiable in established range of variables M and is not a hyper-transcendental function. We shall look for the first-order system of differential equations:

$$\frac{dx_i}{d\varphi} = f_i, \quad i = 1, 2, \ldots, n, \tag{2}$$

the solution of which satisfies the equation (1) in established range of variables M.

The variable φ in (2) is an argument of the designed differential analyzer. In is necessary to define the functions f_i. After differentiation of equation (1) by parameter φ, we shall get:

$$\sum_{i=1}^{n} \frac{\partial F}{\partial x_i} \frac{dx_i}{d\varphi} = 0. \tag{3}$$

If the system (2) solution turns into identity the equation (1), then system (2) turns into identity the equation (3). Thus, the functions f_i definition can be based on analytical condition (1). This problem has a solution set, at that functions f_i in all cases depend on partial derivatives $\frac{\partial F}{\partial x_i}$. For analytical algorithm simplification let us concern the functions f_i are linear functions of mentioned above partial derivatives. This method of differential analyzers synthesis has an essential advantage: the argument φ, which is concerned to be a system parameter, can be any analytical function, what specifically lets realize the argument control, which is necessary for differential analyzer structure simplification.

References

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THE NASH'S OPTIMAL CONTROL OF FOREST ECOSYSTEM

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In [1] was offered the next model 4-tier mosaic forest communities, characterized by productivity x and the soil fertility measure y:

$$\begin{cases} \dfrac{dx}{dt} = -\alpha x^5 - x^3 k - x^2 m - xa - w, \\ \dfrac{dy}{dt} = -\gamma y^3 + \gamma y p - \delta \cdot (w + (w_0 - w\_)), \end{cases} \tag{1}$$

$$0 < w_- < w_0 < w_+,$$

$$t \in [0, T],$$

where m is mosaic state, k is interspecific and intraspecific competition, a is the anthropogenic impact, w is soil moisture, p is the measure of soil type and $\alpha, \gamma, \delta > 0$ are constants. Here $k = 0, a = 0, w = 0$ are the boundaries of ecological stability of phytocenosis, and $x = 0$ is characteristic observed value of productivity in the absence of strong changes in external factors.

Position control $\{u_1^* = m^*(x,y), u_2^* = k^*(x,y), u_3^* = a^*(x,y), u_4^* = w^*(x,y), u_5^* = p^*(x,y)\}$ are said to constitute a *Nash's optimal control* if

$$J_i(u_1^*, u_2^*, u_i^*, ..., u_N^*) \le J_i(u_1^*, u_2^*, ..., u_{i-1}^*, u_i, u_{i+1}^*, ..., u_N^*), \ \ \forall u_i, \ \ i = 1, ..., N, \ N = 5,$$

where

$$J_i(u_1^*, u_2^*, u_i^*, ..., u_N^*) = \int_0^{+\infty} [Q_i(x) + u_i^2] dt.$$

Using [2] we found the Nash's position optimal control

$$k^* = \frac{1}{2}x^4, \quad m^* = \frac{1}{2}x^3, \quad a^* = \frac{1}{2}x^2, \quad w^* = \frac{1}{2}x, \quad p^* = 0$$

for our forest ecosystem model with
$Q_1 = \alpha x^6 + \frac{1}{2}\left(\frac{1}{2}x^8 + x^6 + x^4 + x^2\right), Q_2 = \alpha x^6 + \frac{1}{2}\left(x^8 + \frac{1}{2}x^6 + x^4 + x^2\right),$
$Q_3 = \alpha x^6 + \frac{1}{2}\left(x^8 + x^6 + \frac{1}{2}x^4 + x^2\right), Q_4 = \alpha x^6 + \frac{1}{2}\left(x^8 + x^6 + x^4 + \frac{1}{2}x^2\right),$
$Q_5 = \alpha x^6 + \frac{1}{2}\left(x^8 + x^6 + x^4 + x^2\right).$
For this optimal control productivity x asymptotically tends to zero with $t \to +\infty$, i.e. to characteristic observed value of productivity in region, but dynamics of forest ecosystem is not asymptotically stable. Since $u_i^* > 0$ for $x > 0$, then we have a slowly degrading forest.

References

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DIFFERENTIAL EQUATIONS OF MOTIONS OF MULTI-AXIS SYSTEMS

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The problem of program motion synthesis is generally solved without uniqueness and control functions realizing the motion and minimizing a functional must be obtained.

Differential equations of motions of multi-axis systems based on linear spepping motors [1, 2] can be represented as

$$\dot{x}_i = p_i(\mathbf{x}) + u_i(\mathbf{x})b_i(\mathbf{x}), \quad i = 1, \ldots, n. \tag{1}$$

where $\mathbf{x} = (x_1, \ldots, x_n)$ are generalized device coordinates, $\mathbf{u} = (u_1, \ldots, u_n)$ is the control vector.

The problem consists in forming controls $u = (t, \mathbf{x})$ such that $u \in \mathbf{R}^r$ and corresponding solution of the system (1) satisfies the additional conditions

$$\omega_k(t, \mathbf{x}) = 0, \quad k = 1, \ldots, r. \tag{2}$$

However, if $\mathbf{x} = \mathbf{x}(t)$ is a solution satisfied the program (2) then $\omega_k(t, \mathbf{x}(t)) \equiv 0$, $k = 1, \ldots, r$.

Whence

$$\frac{d}{dt}\omega_k(t, \mathbf{x}(t)) \equiv 0, \quad k = 1, \ldots, r$$

or

$$\sum_{i=1}^{n}\left(\frac{\partial\omega_k(t,\mathbf{x})}{\partial x_i}(p_i(\mathbf{x}) + u_i b_i(\mathbf{x})) + \frac{\partial\omega_k(t,\mathbf{x})}{\partial t}\right) \equiv 0,$$

when $\mathbf{x}$ satisfies (2).

The last expression is equivalent to the condition

$$\sum_{i=1}^{n}\left(\frac{\partial\omega_k(t,\mathbf{x})}{\partial x_i}(p_i(\mathbf{x}) + u_i b_i(\mathbf{x})) + \frac{\partial\omega_k(t,\mathbf{x})}{\partial t}\right) = R_k(t, \mathbf{x}, \omega_k), \quad k = 1, \ldots, r, \tag{3}$$

where R_k is the arbitrary functions such that $R_k(t, \mathbf{x}, 0) \equiv 0$.

Therefore, the condition (3) is neccessary and sufficient for implementing the program (2) along solution $\mathbf{x} = \mathbf{x}(t)$ of system (1). It can be used for calculating the neccessary controls $u_i(t, \mathbf{x})$, $i = 1, \ldots, r$.

As $r < n$, the system (3) defines the controls ambiguously, and the functional must be minimized on free controls additionally. E.g. the control optimization problem with constraints